HEATING OF A NEWTONIAN LIQUID WHEN EXTRUDED THROUGH A MATRIX

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Calculations are carried out of the dynamic forces and viscous heating when a layer of Newtonian liquid at the bottom of a rigid container is extruded through a matrix in the form of a circular opening or a narrow slit.

The extrusion of a material through a matrix in the wall of a container is one of the most widely used technological processes carried out under pressure. A calculation of the necessary forces for this operation for plastic materials has been carried out in a number of papers (reviews are given in [1, 2]). The problem of the rapid extrusion of a thin layer of a nonlinearly viscous medium through a narrow slit or a circular opening of finite length l has not been investigated in great detail. Apart from a prediction of the forces required to achieve this process, it is of interest to analyze the heating that occurs, since it is particularly important to know this when working with explosive materials.

We will obtain the velocity, pressure, and temperature distributions of solving the problem of the extrusion of an incompressible Newtonian liquid through a matrix, assuming the liquid to obey a power rheological law [3]

$$\tau = m|\dot{\gamma}|^{n-1}\dot{\gamma},$$

where m is the index of the Newtonian behavior of the material, and takes the value of the dynamic viscosity μ when $n = 1$ (when $n = 0$ we assume $m = \tau_s$ — the yield point of an ideal-plastic medium to shear).

A physical model of the phenomenon considered and the system of coordinates used are shown in the figure. There is a circular opening of radius $r_0 \ll l$ (or a longitudinal slit of width $2r_0$) along the axis of symmetry of a container with a base radius R (or half-thickness R in the case of plane deformation of the layer). At the initial instant of time the layer of liquid of thickness $\delta_0 \ll R$ is set in motion due to the axial displacement of a rigid instrument (a die) of mass M with velocity $w_0 < 0$. We will assume that there is no friction between the side walls of the container and the die, and that the slowing down of the latter is due exclusively to the action of the viscous forces that occur due to the flow of liquid inside the opening and in the gap between the bottom of the container and the instrument.

We will confine the analysis to the case of a narrow opening $r_0 \ll R$. Then the whole region of flow can be divided into two separate parts — the flow inside the opening and in the gap between the container and the die, in each of which the motion of the liquid will be assumed to be steady-state. In fact, in this case arrangement of the flow from predominantly radial to axial is localized in a region of dimensions $\sim r_0$, where the time of this process $\sim\rho_0 r_0^2 \mu^{-1}$ is small compared with the characteristic extrusion time of the layer $\delta_0|w_0|^{-1}$, if the viscosity of the liquid is fairly high ($\sim 10^3$-10^4 P).

1. We will determine the pressure p_0 for the liquid to flow into the opening. In the case of noninertial motion ($Re = \rho_0 V r_0 \mu^{-1} \ll 1$) the equations of hydrodynamics, describing the flow of the liquid inside the opening, can be written as

$$\frac{\partial p}{\partial z} = \frac{1}{r^k}\frac{\partial}{\partial r}\left(r^k m\left|\frac{\partial V}{\partial r}\right|^{n-1}\frac{\partial V}{\partial r}\right), \quad \frac{\partial p}{\partial r} = 0,$$
$$p(0) = p_0,\ V(r_0) = V(-r_0) = 0, \tag{1}$$

where the index $k = 1$ for a circular opening and $k = 0$ for a plane slit.

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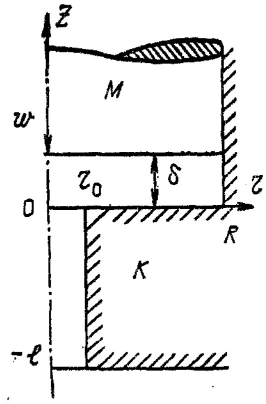

Fig. 1. Sketch of the extrusion of a liquid from a container (K) by the axial displacement of a die (M); r0z is the system of coordinates.

We will represent the velocity of the liquid in the form of the function

$$V = V_m (1 - |r r_0^{-1}|^{(n+1)/n}), \tag{2}$$

which satisfies the conditions of adhesion to the walls of the opening. Substituting V(r) into (1) and using the condition for material balance, we obtain an equation for p(z), by integrating which for m = const, we obtain

$$p_0 = p_a + \left(\frac{n(2+k)+1}{n r_0} \left(\frac{R}{r_0} \right)^{k+1} |w| \right)^n \frac{(k+1) m l}{r_0}, \tag{3}$$

where p_a is the surrounding (atmospheric) pressure at the exit from the opening.

We will now analyze the motion of the liquid in the gap between the container and the die. When $\delta \ll R$ and $\mathrm{Re} = \rho_0 u \delta \mu^{-1} \ll 1$ the equations of hydrodynamics can be written, with an accuracy $\sim(\delta/R)^2$, in the form

$$\begin{gathered} \frac{\partial p}{\partial r} = \frac{\partial}{\partial z} \left(m \left| \frac{\partial u}{\partial z} \right|^{n-1} \frac{\partial u}{\partial z} \right), \quad \frac{\partial p}{\partial z} = 0, \\ \frac{\partial v}{\partial z} = -\frac{1}{r^k} \frac{\partial}{\partial r} (r^k u), \quad p(r_0) = p_0, \\ u(r,0) = u(r,\delta) = u(R,z) = v(r,0) = 0, \; v(r,\delta) = w. \end{gathered} \tag{4}$$

It follows from (4) that the equations of motion and the boundary conditions are satisfied by the following velocity field:

$$\begin{gathered} u = f(r,\delta)(1 - |1 - 2z\delta^{-1}|^{(n+1)/n}), \\ f = \frac{2n+1}{(n+1)(k+1)} \frac{w}{\delta} \left[\left(\frac{R}{r} \right)^k R - r \right], \\ v = \frac{w}{2(n+1)} \left[(2n+1) \frac{2z}{\delta} - n \left(1 \mp \left| 1 - \frac{2z}{\delta} \right|^{(2n+1)/n} \right) \right] \\ (- \text{ for } 0 \leqslant z \leqslant \delta/2; \; + \text{ for } \delta/2 \leqslant z \leqslant \delta). \end{gathered} \tag{5}$$

Substituting u(r, z) into the equation of motion (4) in the projection onto the r axis, we obtain the pressure distribution at the end of the punch

$$\begin{gathered} p = p_0 + \Pi I_1(\xi), \quad \xi = r/R, \quad \xi_0 = r_0/R, \\ \Pi = \left(\frac{2}{k+1} \frac{2n+1}{n} \right)^n \frac{2m|w|^n R^{n+1}}{\delta^{2n+1}}, \\ I_1(\xi) = \int_{\xi_0}^{\xi} (\xi^{-k} - \xi)^n \, d\xi \end{gathered} \tag{6}$$

and the average pressure acting on the layer from the side of the die

$$\langle p \rangle = p_0 + \frac{(k+1)\Pi I_2}{(1-\xi_0)(1+\xi_0)^k}, \quad I_2 = \int_{\xi_0}^{1} I_1(\xi) \xi^k \, d\xi. \tag{7}$$

For plane deformation of the layer we obtain values of the integrals and the specific compressing load on the instrument

$$I_1(\xi) = \frac{(1-\xi_0)^{n+1} - (1-\xi)^{n+1}}{n+1}; \quad I_2 = \frac{(1-\xi_0)^{n+2}}{n+2},$$
$$\langle p \rangle = p_0 + \frac{\Pi_{k=0}}{n+2}(1-\xi_0)^{n+1}. \tag{8}$$

In the case of axisymmetrical deformation, integrals I_1 and I_2 can be evaluated only for integer values of n (including $n=0$). For arbitrary n they can easily be found by any numerical methods.

Using the differential form of Newton's second law, we obtain the following equation describing the slowing down of an absolutely rigid die:

$$Mw\,dw/d\delta = (\langle p \rangle - p_a)\,S,$$
$$S = (\pi R)^k R\,(2a)^{1-k}, \quad w(\delta_0) = w_0. \tag{9}$$

In dimensionless variables $x = \delta_0/\delta$ and $y = w/w_0$, the solution of (9) can be written as

$$y = [1 - (2-n)\,\beta\,((x^{2n} - 1)/2n + A(1 - x^{-1}))]^{1/(2-n)},$$
$$\beta = SA_1\delta_0(Mw_0^2)^{-1}, \quad A = A_1A_2^{-1},$$
$$A_1 = \left(\frac{2}{k+1}\,\frac{2n+1}{n}\right)^n \frac{2m(k+1)R^{n+1}|w_0|^n I_2}{(1-\xi_0)(1+\xi_0)^k\,\delta_0^{2n+1}},$$
$$A_2 = \left(\frac{n(2+k)+1}{n\xi_0^{k+1}}\right)^n \frac{(k+1)ml|w_0|^n}{r_0^{n+1}}. \tag{10}$$

When compressing a layer with a constant force $F = (\langle p \rangle - p_a)S$ the equation representing the slowing down of the die has the form

$$y = [\beta_1(x^{2n+1} + A)^{-1}]^{1/2n}, \quad \beta_1 = F/SA_1. \tag{11}$$

2. We will obtain equations for the dissipative heating of the liquid in the gap between the container and the die. For flow with large values of $Pe = \rho_0 c_p |w| \delta \lambda_0^{-1} \gg 1$ (a non-heat-conducting liquid) the heat-balance equation can be written in the form

$$\rho_0 c_p \left(\frac{d\theta_1}{dt}\right)_{r^0,z^0} = m\left|\frac{\partial u}{\partial z}\right|_{r,z}^{n+1},$$
$$\left(\frac{\partial r}{\partial t}\right)_{r^0,z^0} = u, \quad \left(\frac{\partial z}{\partial t}\right)_{r^0,z^0} = v,$$
$$r(r^0, z^0, 0) = r^0, \quad z(r^0, z^0, 0) = z^0,$$
$$\theta_1(r^0, z^0, 0) = 0, \tag{12}$$

where the upper index 0 denotes the Lagrangian coordinates of the particles of the medium at the initial instant of time $t = 0$.

It follows from (5) that the maximum heating of the liquid is observed at the point of discharge $r = r_0$ and $z = 0$, $\delta/2$. In order to avoid discontinuous solutions when $n = 0$, we will obtain an equation for the average temperature of the liquid over the thickness of the layer at the point of maximum $r = r_0$

$$\langle \theta_1 \rangle = \left(\frac{2n+1}{n(k+1)}\right)^n \frac{2^{n+1}m|w_0|^n R^{n+1}}{(k+1)\rho_0 c_p \delta_0^{2n+1}} (\xi_0^{-k} - \xi_0)^{n+1} I_3(x),$$
$$I_3(x) = \int_1^x y^n x^{2n} dx. \tag{13}$$

Substituting (10) into (13) we can obtain the variation of $\langle\theta_1\rangle(x)$ for arbitrary action over the layer of liquid material. The following methods of processing under pressure are of interest: compression of a layer at constant velocity and extrusion with a constant force. In the first case $y \equiv 1$ and $I_3 = (2n+1)^{-1}(x^{2n+1} - 1)$, and in the second we use Eq. (11) and obtain

$$I_3 = \beta_1 (2n+1)^{-1} \ln [(x^{2n+1} + A)(1+A)^{-1}].$$

To calculate the adiabatic heating of the liquid for flow inside an opening we return to the equation of inflow of heat

$$\rho_0 c_p \frac{d\theta_2}{dt} = m \left| \frac{\partial V}{\partial r} \right|^{n+1}, \quad \theta_2(0) = \langle \theta_1 \rangle . \tag{14}$$

If follows from Eq. (2) for the velocity distribution V(r) that maximum liquid heating occurs at the exit from the opening $z = -l$ at points $r = \pm r_0$. For $n = 0$ the maximum temperature is indeterminate. Hence, we will derive an equation for the average heating of the liquid over the cross section of the opening at the point of maximum $z = -l$, using the expression relating the level of the liquid in the opening with the flow of the liquid layer $|l| = \delta_0 \xi^{-(k+1)}(1 - x^{-1})$. We finally obtain

$$\langle \theta_2 \rangle = \langle \theta_1 \rangle + H I_4(x), \quad I_4(x) = \int_1^x y^n x^{-2} dx,$$

$$H = \left(\frac{n(2+k)+1}{n} \right)^n \frac{2^k m \delta_0 |w_0|^n}{\rho_0 c_p r_0^{2n+1}} \xi_0^{-(n+1)(k+1)}. \tag{15}$$

Let us consider some special methods of processing the layer. For compression with a constant velocity $I_4 = 1 - x^{-1}$; for extrusion with constant force

$$I_4 = \beta_1 \int_1^x x^{-2} (x^{2n+1} + A)^{-1} dx.$$

This integral can be evaluated when $n = 0$ and gives

$$A^{-1}(1 - x^{-1}) + A^{-2} \ln [(x + A) x^{-1} (1 + A)^{-1}].$$

We will analyze the relations obtained for dissipative heating of the material.

As can be seen from (13) and (15), the liquid is more strongly heated when there is axial symmetry of the flow than when there is plane symmetry. This is due to the convergence of the radial flow, in which case the energy of motion gradually accumulates at the point where the liquid material discharges.

For example, in the case of compression at constant velocity $|w_0| = 1$ m/sec of a pseudoplastic material with $n = 0.5$, $m = 10^3$ Pa$\cdot$sec$^{1/2}$, $\rho_0 c_p = 2.0 \cdot 10^6$ J/m$^3\cdot$deg K, in the form of a layer $\delta_0 = 2 \cdot 10^{-3}$ m in a container with $r_0 = 10^{-3}$ m and $R = 10^{-2}$ m, we find when $x = 10$ values of $\langle\theta_1\rangle = 385$°K for $k = 1$ and 30°K for $k = 0$.

We will compare the maximum heating for the inflow and outflow of a liquid from an opening when the layer is compressed with constant velocity

$$\frac{\langle \theta_1 \rangle}{\langle \theta_2 \rangle} = 1 + \frac{(2n+1)(k+1)^{(n+1)}}{2^{n+1-k}} \left(\frac{n(2+k)+1}{2n+1} \right)^n \left(\frac{\delta_0}{r_0} \right)^{2(n+1)} \sigma(x),$$

$$\sigma(x) = (x - 1)/x (x^{2n+1} - 1).$$

The function $\sigma(x)$ decreases monotonically from its greatest value $(2n+1)^{-1}$ for $x = 1$ to 0 as $x \to \infty$. Hence, at the beginning of the compression the ratio between $\langle\theta_2\rangle$ and $\langle\theta_1\rangle$ as a whole is determined by the ratio δ_0 / r_0. When $\delta_0 > r_0$ the heating of the liquid is primarily due to the flow inside the opening. For high degrees of compression the layer of liquid is mainly heated due to energy dissipation in the gap between the die and the container.

In conclusion, we will compare the maximum heating in converging and diverging flows of a liquid material. In the first case we use Eq. (13), and in the second we use the results of a solution of the hydrodynamic problem of the free flow of a layer of an incompressible Newtonian liquid when a load is applied axially [4] (there is no central opening and no side walls). It is shown in [4] that for a diverging flow geometry the temperature maximum of a non-heat-conducting liquid is reached at the periphery of the die $r = R$. Using the results obtained in [4] or Eq. (5) directly, in which $f(r, \delta)$ must be put equal to $(2n+1)|w|R/(n+1)(k+1)\delta$, we obtain, other conditions being equal, that the ratio of these temperatures is

$$\zeta=\frac{\langle\theta_1\rangle_{r_0=0}}{\langle\theta_1\rangle_{r_0\neq0}}=\left(\frac{\xi_0^k}{1-\xi_0^{k+1}}\right)^{n+1}.$$

For plane deformation of the layer ($k=0$) the maximum heating in both cases is approximately the same. A different picture ($\zeta \ll 1$) is observed in the case of axisymmetrical deformation, since $\xi_0 \ll 1$. The result obtained is a consequence of the accumulation of energy in the converging flow of viscous material.

NOTATION

r, z, axes of a Cartesian ($k=0$) or a cylindrical ($k=1$) system of coordinates; u, v, velocity components of the liquid in the directions r and z, respectively; V, velocity of the liquid in the opening; p, pressure; θ_1 and θ_2, temperature of the liquid under the die and in the opening; m, n, rheological constants of the material; τ, tangential stress; $\dot{\gamma}$, shear velocity; ρ_0, c_p, λ_0, density, heat capacity, and thermal conductivity of the liquid; S, M, w, area of the working surface, mass and velocity of motion of the die; r_0, l, a, R, linear dimensions of the matrix and the container; F, operating force on the instrument; Re, Pe, similitude criteria.

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